ΛΥΣΕΙΣ ΔΙΑΓΩΝΙΣΜΑΤΟΣ 4ου ΚΕΦ

ΘΕΜΑ Α

Α1) → δ Α2) → δ Α3) → β Α4) → β

Α5) α → Σ β → Σ γ → Λ δ → Λ ε → Λ

ΘΕΜΑ Β

Β1) α) Σωστή απάντηση η (iii)

β) $I_x = I_{cm} + m\left(\frac{L}{2} - a\right)^2$ (1)

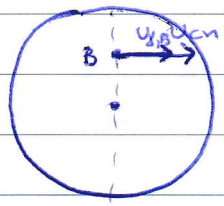
$I_y = I_{cm} + m\left(\frac{L}{2} - b\right)^2$ (2)

$a > b \Rightarrow \left(\frac{L}{2} - a\right)^2 < \left(\frac{L}{2} - b\right)^2$ (3)

(1), (2), (3) $\Longrightarrow I_x < I_y \Rightarrow \frac{I_x}{I_y} < 1$

Β2) α) Σωστή απάντηση η (i)

β)



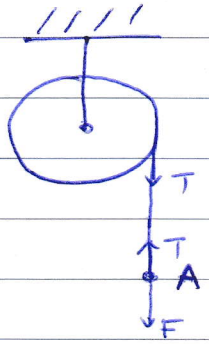
$U_{cm} = \omega R$ (1) $U_{\gamma,B} = \omega \cdot \frac{R}{2}$ (2)

$\vec{U_B} = \vec{U_{cm}} + \vec{U_{\gamma,B}} \Rightarrow U_B = U_{cm} + U_{\gamma,B} \Rightarrow$

$U_B = \omega R + \frac{\omega R}{2} \Rightarrow U_B = \frac{3\omega R}{2} \Rightarrow U_B = \frac{3}{2} U_{cm}$

Β3) α) Σωστή απάντηση η (ii)

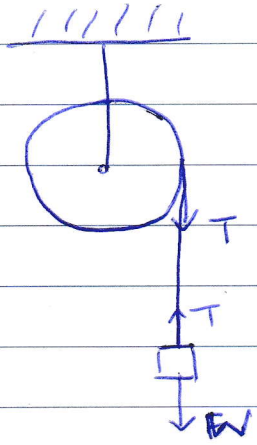
β)



Σημείο Α: $\Sigma F = 0 \Rightarrow F = T$

Τροχαλία: $\Sigma \tau = I \alpha_{\gamma_1} \Rightarrow T \cdot R = I \cdot \alpha_{\gamma_1} \Rightarrow$

$F \cdot R = I \cdot \alpha_{\gamma_1} \Rightarrow \alpha_{\gamma_1} = \frac{F \cdot R}{I}$ (1)



Σώμα: $\Sigma F = m \cdot a \Rightarrow W - T = m \cdot a \xrightarrow[a = a_{\gamma 2} R]{F = W}$

$mg - T = m \cdot a_{\gamma 2} \cdot R \quad (2)$

Τροχαλία: $\Sigma \tau = I \cdot a_{\gamma 2} \Rightarrow T \cdot R = I a_{\gamma 2} \Rightarrow$

$T = \frac{I \cdot a_{\gamma 2}}{R} \quad (3)$

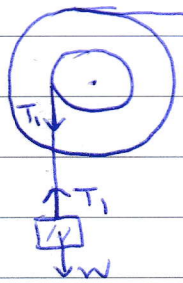
$(2)+(3) \Rightarrow mg = m \cdot R a_{\gamma 2} + \frac{I a_{\gamma 2}}{R} \Rightarrow mg = \left(mR + \frac{I}{R}\right) a_{\gamma 2} \Rightarrow$

$mg = \frac{mR^2 + I}{R} \cdot a_{\gamma 2} \Rightarrow a_{\gamma 2} = \frac{F \cdot R}{mR^2 + I} \quad (4)$

$\frac{(1)}{(4)} \Rightarrow \frac{a_{\gamma 1}}{a_{\gamma 2}} = \frac{\frac{F \cdot R}{I}}{\frac{F \cdot R}{mR^2 + I}} \Rightarrow \frac{a_{\gamma 1}}{a_{\gamma 2}} = \frac{mR^2 + I}{I} > 1 \Rightarrow a_{\gamma 1} > a_{\gamma 2}$

ΘΕΜΑ Γ

Γ1)



Σώμα Σ ισορροπεί: $\Sigma F = 0 \Rightarrow$

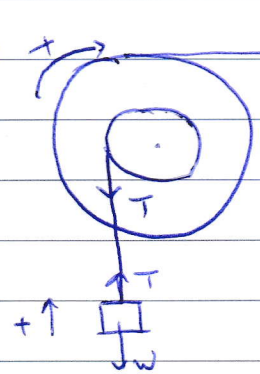
$T_1 - W = 0 \Rightarrow T_1 = mg \Rightarrow T_1 = 200\,N$

Στερεό Π ισορροπεί: $\Sigma \tau = 0 \Rightarrow$

$F_0 \cdot 2R - T_1 \cdot R = 0 \Rightarrow 2F_0 R = T_1 R \Rightarrow F_0 = \frac{T_1}{2} \Rightarrow$

$\boxed{F_0 = 100\,N}$

Γ2)



Σώμα Σ: $\Sigma F = m \cdot a \Rightarrow T - W = ma \Rightarrow$

$T - m \cdot g = ma \quad (1)$

Στερεό Π: $\Sigma \tau = I \cdot a_\gamma \Rightarrow F \cdot 2R - T \cdot R = MR^2 a_\gamma$

$\xrightarrow{a = a_\gamma R}$ $2F - T = Ma \quad (2)$

$(1)+(2) \Rightarrow 2F - mg = ma + Ma \Rightarrow$

$230 - 200 = 30a \Rightarrow \boxed{a = 1\,m/s^2}$

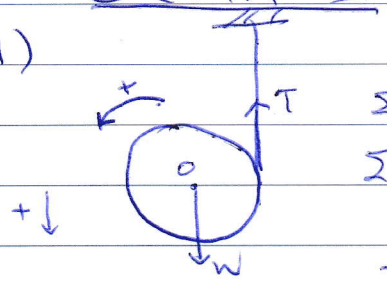
Γ3) Σώμα Σ: $h = \frac{1}{2} a t^2 \Rightarrow t = \sqrt{\frac{2h}{a}} \Rightarrow t = 2\,s$

Στερεό Π: $\omega = a_\gamma \cdot t \Rightarrow \omega = \frac{a}{R} t \Rightarrow \boxed{\omega = 10\,r/s}$

Γ4) $a_A = a_\gamma \cdot 2R \Rightarrow a_A = \frac{a}{R} \cdot 2R \Rightarrow a_A = 2 m/s^2$

$\Delta x_A = \frac{1}{2} a_A t^2 \Rightarrow \boxed{\Delta x_A = 4m}$

ΘΕΜΑ Δ



Δ1) $a_{cm} = a_\gamma R$ (1)

$\Sigma F_y = m \cdot a_{cm} \Rightarrow mg - T = m \cdot a_{cm}$ (2)

$\Sigma \tau_{(O)} = I \cdot a_\gamma \Rightarrow T \cdot R = \frac{1}{2} m R^2 \cdot a_\gamma \overset{(1)}{\Longrightarrow}$

$T = \frac{1}{2} m a_{cm}$ (3)

(2) + (3) $\Rightarrow mg = m a_{cm} + \frac{1}{2} m a_{cm} \Rightarrow a_{cm} = \frac{2g}{3} \Rightarrow \boxed{a_{cm} = \frac{20}{3} m/s^2}$

(1) $\Rightarrow \boxed{a_\gamma = \frac{200}{3} r/s^2}$

Δ2) (3) $\Rightarrow T = \frac{1}{2} \cdot \frac{20}{3} \Rightarrow \boxed{T = 20N}$

Δ3) $l = x \Rightarrow l = \frac{1}{2} a_{cm} t^2$ (4)

$v_{cm} = a \cdot t_1 \Rightarrow t_1 = \frac{2}{20/3} \Rightarrow t_1 = 0,3s$

(4) $\Rightarrow \boxed{l = 0,3m}$

Δ4) Από $t = 0$ έως $t_1 = 0,3s$ το στερεό εκτελεί μεταφορική ε.ο.επιτ. κίνηση με $a_{cm} = 20/3 \, m/s^2$ και στροφική ομαλά επιταχυνόμενη με $a_\gamma = \frac{200}{3} r/s^2$

Την $t_1 = 0,3s$ $v_1 = 2 m/s$ και $\omega_1 = \frac{v_1}{R} = 20 r/s$

Από την $t_1 = 0,3s$ έως $t_2 = 1,1s$ αφήνεται μόνο το βάρος δηλ. $\Sigma F_y = W$ και $\Sigma \tau = 0$ άρα το στερεό εκτελεί μεταφορικά ε.ο.επιτ με $a_{cm} = g = 10 m/s^2$ και στροφική ομαλή κίνηση

$t_2 = 1,1s$: $\omega_2 = \omega_1 = 20 r/s$

$v_2 = v_1 + g(t_2 - t_1) \Rightarrow v_2 = 10 m/s$

Δ4 i)