ΑΣΚΗΣΗ 1η

α) $t=0 \; ; \; x=+A$

$x = A\eta\mu(\omega t+\phi_0) \xrightarrow[x=+A]{t=0} \eta\mu\phi_0 = 1 \xrightarrow{0\le\phi_0<2\pi} \boxed{\phi_0 = \frac{\pi}{2}\ rad}$

β) $x = A\eta\mu(\omega t+\phi_0) \xrightarrow{t=\frac{T}{6}} x = 0{,}4\,\eta\mu\left(\frac{2\pi}{T}\cdot\frac{T}{6}+\frac{\pi}{2}\right) \Rightarrow$

$x = 0{,}4\,\eta\mu\frac{5\pi}{6} \Rightarrow x = 0{,}4\cdot\frac{1}{2} \Rightarrow \boxed{x = 0{,}2\,m}$

γ) $k = (m_1+m_2)\omega^2 \Rightarrow \omega' = \sqrt{\frac{400}{4}} \Rightarrow \omega' = 10\ r/s$

$\omega' = \frac{2\pi}{T'} \Rightarrow T' = \frac{2\pi}{10} \Rightarrow \boxed{T = \frac{\pi}{5}\ sec}$

δ) $k = m_1\omega^2 \Rightarrow \omega = \sqrt{\frac{400}{3}} \Rightarrow \omega = \frac{20}{\sqrt{3}}$

$v_{max} = \omega A \Rightarrow v_{max} = \frac{8}{\sqrt{3}}$

$v_1 = v_{max}\,\sigma\upsilon\nu(\omega t+\phi_0) \Rightarrow v_1 = \frac{8}{\sqrt{3}}\sigma\upsilon\nu\frac{5\pi}{6} \Rightarrow v_1 = \frac{8}{\sqrt{3}}\left(-\frac{\sqrt{3}}{2}\right)$

$v_1 = -4\ m/s$

ΑΔΟ : $m_1|v_1| + m_2|v_2| = (m_1+m_2)V$

$3\cdot 4 + 1\cdot 8 = 4V$

$V = 5\ m/s$

Α.Δ.Ε.Τ : $E_T = K + U$

$E_T = \frac{1}{2}(m_1+m_2)V^2 + \frac{1}{2}kx^2$

$E_T = \frac{1}{2}4\cdot 5^2 + \frac{1}{2}400\cdot 0{,}2^2$

$E_T = 58\ J$

ΑΣΚΗΣΗ 2η

Δ1) Α.Δ.Ε.Τ για Σ_1 πριν κρούση

$$E_T = K + U \Rightarrow \frac{1}{2}KA^2 = \frac{1}{2}Mv_1^2 + \frac{1}{2}kx^2$$

$$100 \cdot 0,04 = 3v_1^2 + 100 \cdot 0,01$$

$$v_1 = 1\ m/s$$

Δ2) Α.Δ.Ο : $M \cdot v_1 = (M+m)V \Rightarrow V = \frac{3}{4}\ m/s$

$$K_{αρχ} = \frac{1}{2}Mv_1^2 = \frac{3}{2}\ J$$

$$K_{τελ} = \frac{1}{2}(M+m)V^2 = \frac{1}{2}\,4\,\frac{9}{16} = \frac{9}{8}\ J$$

$$|\Delta K| = |K_{τελ} - K_{αρχ}| = \frac{3}{8}$$

$$\Pi = \frac{|\Delta K|}{K_{αρχ}} \cdot 100\% \Rightarrow \Pi = \frac{\frac{3}{8}}{\frac{3}{2}} \cdot 100\% \Rightarrow \Pi = 25\%$$

Δ3) Α.Δ.Ε.Τ για συσσωμάτωμα

$$E_T = K + U$$

$$\frac{1}{2}kA^2 = \frac{1}{2}(M+m)V^2 + \frac{1}{2}kx^2$$

$$100A^2 = 4 \cdot \frac{9}{16} + 100 \cdot 0,01$$

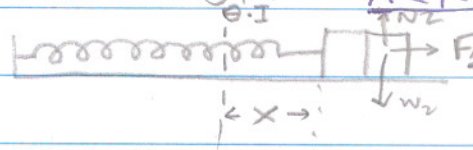
$$100A^2 = \frac{13}{4} \Rightarrow A = \frac{\sqrt{13}}{20}\ m$$

Δ4) $$\left|\frac{\Delta K}{\Delta t}\right| = \left|\frac{\Sigma F \cdot \Delta x}{\Delta t}\right| = |\Sigma F \cdot V| = k \cdot x \cdot V$$

$$= 100 \cdot 0,1 \cdot \frac{3}{4} = 7,5\ J/s$$

ΑΣΚΗΣΗ 3

Δ1)



Σ_2 : $\Sigma F_x = -D_2 \cdot x \Rightarrow F_1 = -D_2 \cdot x$ (1)

Χάνει την επαφή του όταν $F_1 = 0$ Άρα (1) $\Rightarrow x = 0$

Δ2) Συσσωμάτωμα : $A = d = 0,4m$

$K = (m_1 + m_2)\omega^2 \Rightarrow \omega = \sqrt{\frac{100}{4}} \Rightarrow \omega = 5 rad/s$

$U_{max} = \omega A \Rightarrow U_{max} = 2 m/s$

Το Σ_2 χάνει τη επαφή με το Σ_1 όταν Θ.Ι $\equiv$ ΘΦΜ

Το Σ_1 μετά το χάσιμο επαφής εκτελεί Α.Α.Τ. ξεκινώντας από το ΘΦΜ $\equiv$ Θ.Ι με $U = 2 m/s$

Άρα $U_{max_1} = U_{max} \Rightarrow U_{max_1} = 2 m/s$

Σ_1 : $k = m_1 \omega_1^2 \Rightarrow \omega_1 = \sqrt{\frac{100}{1}} \Rightarrow \omega_1 = 10 rad/s$

$U_{max,1} = \omega_1 A_1 \Rightarrow A_1 = \frac{2}{10} \Rightarrow A_1 = 0,2m$

Δ3) Α.Δ.Ο. : $m_2 U_{max} = (m_2 + m_3) \cdot V \Rightarrow$

$V = \frac{3 \cdot 2}{5} \Rightarrow V = 1,2 m/s$

Δ4) $K_{πριν} = \frac{1}{2} m_2 U_{max}^2 = \frac{1}{2} 3 \cdot 2^2 = 6 J$

$K_{μετα} = \frac{1}{2}(m_2 + m_3) V^2 = \frac{1}{2} 5 \cdot \frac{36}{25} = 3,6 J$

$Q = |\Delta K| = 6 - 3,6 = 2,4 J$

$\Pi = \frac{Q}{K_{πριν}} \cdot 100\% = \frac{2,4}{6} \cdot 100\% = 40\%$

ΑΣΚΗΣΗ 4η

α) Α.Δ.Μ.Ε για m_2 πριν κρούση

$K_{αρχ} + U_{αρχ} = K_T + U_T$

$m_2 g h = \frac{1}{2} m_2 U_2^2 \Rightarrow U_2 = \sqrt{2gh} \Rightarrow U_2 = 8 m/s$

β) Α.Δ.Ο : $m_2 U_2 = (m_1 + m_2) \cdot V \Rightarrow V = \frac{1 \cdot 8}{8} \Rightarrow V = 1 m/s$

γ) $\Theta.Ι(m_1) = W_1 = F_{ελ,1} \Rightarrow m_1 g = k \cdot \Delta l_1 \Rightarrow \Delta l_1 = 0,7 m$

$\Theta.Ι(m_1 + m_2) = W_{12} = F_{ελ,2} \Rightarrow (m_1 + m_2) g = k \Delta l_2 \Rightarrow \Delta l_2 = 0,8 m$

$x = \Delta l_2 - \Delta l_1 \Rightarrow x = 0,1 m$

Α.Δ.Ε.Τ για συσσωμάτωμα $E_T = K + U \Rightarrow$

$\frac{1}{2} k A^2 = \frac{1}{2} (m_1 + m_2) V^2 + \frac{1}{2} k x^2 \Rightarrow$

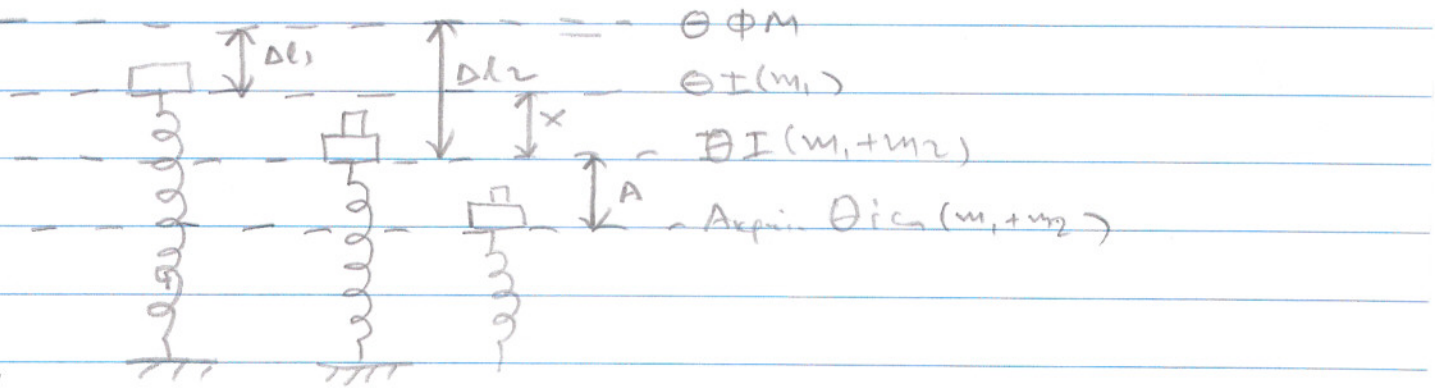
$100 A^2 = 8 \cdot 1^2 + 100 \cdot 0,1^2 \Rightarrow A = 0,3 m$

δ) $U_{ελ,max} = \frac{1}{2} k x_{max}^2$ (1)

x_{max} στην κάτω ακραία θέση Α.ρ. $x_{max} = \Delta l_2 + A$

$x_{max} = 1,1 m$

(1) $\Rightarrow U_{ελ,max} = \frac{1}{2} 100 \, 1,1^2 \Rightarrow U_{ελ,max} = 60,5 J$



ΑΣΚΗΣΗ 5

Γ1) $\Theta.I\,(\Sigma_1): W_1 = F_{ελ,1} \Rightarrow m_1 g = k\Delta\ell \Rightarrow$

$k = \frac{1\cdot 10}{0,05} \Rightarrow k = 200\,N/m$

$\Theta.I\,(\Sigma_1,\Sigma_2): W_{1,2} = K\cdot\Delta\ell' \Rightarrow (m_1+m_2)g = k\Delta\ell'$

$\Delta\ell' = 0,1\,m$

Επειδή ΘΦΜ ≡ Ακραία θέση άρα $A = \Delta\ell' \Rightarrow A = 0,1\,m$

Γ2) $x = \Delta\ell' - \Delta\ell \Rightarrow x = 0,05\,m$

Α.Δ.Ε.Τ για συσσωματώματος

$E_T = K + U \Rightarrow \frac{1}{2}kA^2 = \frac{1}{2}(m_1+m_2)V^2 + \frac{1}{2}kx^2$

$200\cdot 0,01 = 2V^2 + 200\cdot 0,0025$

$V^2 = 0,75 \Rightarrow V = \frac{\sqrt{3}}{2}\,m/s$

Α.Δ.Ο: $m_2 U_0 = (m_1+m_2)V \Rightarrow U_0 = \sqrt{3}\,m/s$

$K_2 = \frac{1}{2}m_2 U_0^2 \Rightarrow K_2 = 1,5\,J$

Γ3) $\vec{\Delta P_2} = \vec{P_{2τελ}} - \vec{P_{2,αρχ}} \overset{\uparrow(+)}{\Longrightarrow} \Delta P_2 = m_2 V - m_2 U_0$

$\Delta P_2 = -\frac{\sqrt{3}}{2}\,kg\cdot m/s$

μέτρο: $\frac{\sqrt{3}}{2}\,kg\,m/s$ κατεύθυνση: αντίθετη της U_0 δηλ. (−)

Γ4) $k = (m_1+m_2)\omega^2 \Rightarrow \omega = \sqrt{\frac{200}{2}} \Rightarrow \omega = 10\,r/s$

$x = A\,ημ(\omega t + \phi_0) \xrightarrow[x=0,05]{t=0} 0,05 = 0,1\,ημ\phi_0 \Rightarrow$

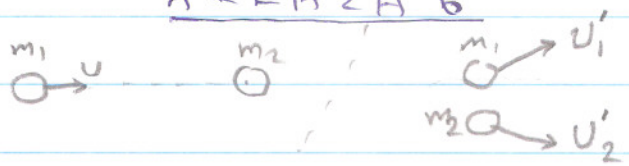
$ημ\phi_0 = \frac{1}{2} \Rightarrow ημ\phi_0 = ημ\frac{\pi}{6}$ (1)

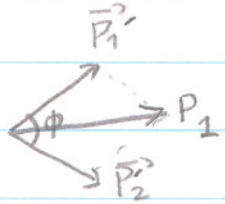
Επειδή $0 \le \phi_0 < 2\pi$ και για $t=0$ $U>0$ άρα δηλ. $\phi_0 > 0$ επομένως (1) $\Rightarrow \phi_0 = \frac{\pi}{6}\,rad$

Άρα $x = 0,1\,ημ(10t + \frac{\pi}{6})$ (S.I.)

ΑΣΚΗΣΗ 6

Δ1)





Ελαστική κρούση: $K_{πριν} = K_{μετα} \Rightarrow$

$$\frac{1}{2} m_1 \upsilon^2 = \frac{1}{2} m_1 \upsilon_1'^2 + \frac{1}{2} m_2 \upsilon_2'^2$$

$$\upsilon^2 = \upsilon_1'^2 + \upsilon_2'^2 \quad (1)$$

Α.Δ.Ο : $\vec{P}_{πριν} = \vec{P}_{μετα} \Rightarrow \vec{P_1} = \vec{P_1}' + \vec{P_2}' \Rightarrow P_1^2 = P_1'^2 + P_2'^2 + 2P_1'P_2' \sigma\upsilon\nu\phi$

$$m_1^2 \upsilon^2 = m_1^2 \upsilon_1'^2 + m_2^2 \upsilon_2'^2 + 2 m_1 \upsilon_1' m_2 \upsilon_2' \sigma\upsilon\nu\phi$$

$$\upsilon^2 = \upsilon_1'^2 + \upsilon_2'^2 + 2\upsilon_1'\upsilon_2' \sigma\upsilon\nu\phi \xrightarrow{(1)}$$

$$\upsilon^2 = \upsilon^2 + 2\upsilon_1'\upsilon_2' \sigma\upsilon\nu\phi \Rightarrow 2\upsilon_1'\upsilon_2' \sigma\upsilon\nu\phi = 0 \Rightarrow$$

$$\sigma\upsilon\nu\phi = 0 \Rightarrow \phi = 90^\circ$$

Δ2) (1) $\xrightarrow[\upsilon_2' = \frac{\upsilon_1'}{\sqrt{3}}]{\upsilon = \frac{4}{3}}$ $\frac{16}{9} = \upsilon_1'^2 + \frac{\upsilon_1'^2}{3} \Rightarrow \frac{16}{9} = \frac{4\upsilon_1'^2}{3} \Rightarrow$

$$\upsilon_1' = \frac{2\sqrt{3}}{3} \text{ m/s} \quad / \quad \upsilon_2' = \frac{\upsilon_1'}{\sqrt{3}} \Rightarrow \upsilon_2' = \frac{2}{3} \text{ m/s}$$

Δ3) Α.Δ.Ο στον άξονα x

$$m_1 \upsilon_{1x} = (m_1 + M) V \Rightarrow m_1 \upsilon_1 \sigma\upsilon\nu\vartheta = (m_1 + M) V \Rightarrow$$

$$1 \cdot \frac{2\sqrt{3}}{3} \cdot \frac{\sqrt{3}}{2} = 4V \Rightarrow V = \frac{1}{4} \text{ m/s}$$

$$K_{πριν} = \frac{1}{2} m_1 \upsilon_1^2 = \frac{1}{2} \cdot 1 \cdot \frac{4}{3} = \frac{2}{3} \text{ J}$$

$$K_{μετα} = \frac{1}{2} (m_1 + M) V^2 = \frac{1}{2} 4 \cdot \frac{1}{16} = \frac{1}{8} \text{ J}$$

$$\Delta K = K_τ - K_{πριν} = \frac{1}{8} - \frac{2}{3} = -\frac{13}{24} \text{ J}$$

Δ4) Θ.Ι.(Μ) : $W_x = F_{ελ} \Rightarrow M \cdot g \cdot ημθ = k \cdot Δl$

$$Δl = \frac{3 \cdot 10 \cdot \frac{1}{2}}{100} \Rightarrow Δl = 0,15\ m$$

Θ.Ι.(Μ+m₁) : $W_{ολ} = F_{ελ}' \Rightarrow (M+m) g \cdot ημθ = kΔl'$

$$Δl' = \frac{4 \cdot 10 \cdot \frac{1}{2}}{100} \Rightarrow Δl' = 0,2\ m$$

$$x = Δl' - Δl \Rightarrow x = 0,05\ m$$

Α.Δ.Ε.Τ για συσσωμάτωμα $E_T = K + U$

$$\frac{1}{2} K A^2 = \frac{1}{2}(m_1 + M) V^2 + \frac{1}{2} K x^2$$

$$100 A^2 = 4 \cdot \frac{1}{16} + 100 \cdot 0,0025 \Rightarrow 100 A^2 = \frac{1}{4} + \frac{1}{4} \Rightarrow$$

$$100 A^2 = \frac{1}{2} \Rightarrow A = \frac{1}{10\sqrt{2}} \Rightarrow A = \frac{\sqrt{2}}{20} = 0,05\sqrt{2}\ m$$